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Study of Beamforming Methods with Steering Vector Errors

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Abstract: Robust Adaptive Beam forming algorithms face many insidious issues such as array steering vector errors which can significantly alter the array gain expected through beam forming. Other issues include coherent signal and interferers, problems like small sample size broadband interferences and so on. Therefore, there is a need to explore and investigate such beam forming methods that solve such RAB issues and are robust to error sources. Furthermore, A performance comparison of sub RAB algorithms should be scrutinized by selecting appropriate performance metrics.

Keywords: Adaptive arrays, Signal mismatch problem, Robust adaptive Beamforming .

I.INTRODUCTION

In the recent decades, adaptive beam forming algorithms have been widely explored and used in various wireless communication technologies such as sonar, and radar communication [1] which is tantamount to saying that considerable number of literature is available for devising robust adaptive beam forming algorithms (RAB). Robustness means to have an adaptive beam formers that can achieve high performance with imperfect, erroneous, and no knowledge about the source, propagation and antenna array. The design of such beam formers assumes that no components of the desired signal are present in the beam formers training data.

Many existing RAB algorithms are in their infancy and face many hazardous issues such as antenna calibration errors [2-6], steering vector errors, and others as robustness against small sample size, robustness against coherent signal and interferers, robustness against coherent signal and interferers, robustness against imperfect waveform coherence at sensor output, robustness against moving and broadband interferences. The traditional approach to the design of adaptive beam forming is to maximize the beam former output signal-to-interference-plus-noise ratio (SINR) assuming that there is no desired signal in the beam forming training data. Although such desired signal free data assumption may be relevant to certain radar applications, the beam forming training snapshots include the desired signal component in most of the practical applications of interest.

In such non-ideal situation, the SINR performance of adaptive beamforming can severely degrade even in the presence of small signal steering vector errors/mismatches, because the desired signal component in the beamformer training data set can be mistakenly interpreted by the adaptive beamforming algorithm as an interferer and, consequently, it can be suppressed rather than being protected. The particular design principles for MVDR RAB include the sidelobe canceller approach, the worst-case optimization and the outage probability constrained optimization, one-dimensional and multi-dimensional covariance fitting, eigen values beamforming using a multi-rank MVDR beamformer and subspace selection, and Robustness is typically understood as an ability adaptive beamforming algorithm to achieve high performance in the situations with imperfect, incomplete, or erroneous knowledge about the source, propagation media, and antenna array. It is also desired to achieve performance with as little possible prior information. In the last decade, several fruitful principle to minimum variance distortionless response (MVDR) robust adaptive beamforming (RAB) design have been developed and succcessfully applied to solve a number of problems in a wide range of applications.

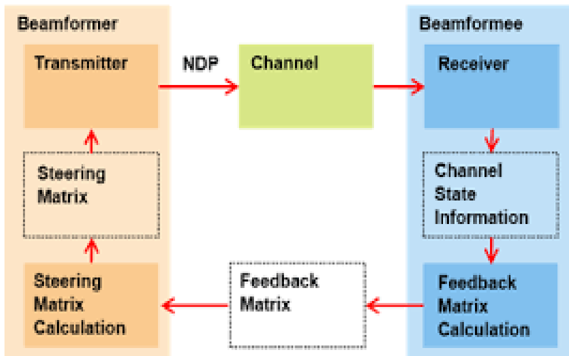

Fig.1 Block diagram Beamforming and steering vector error

Capon beamforming is one of the most useful and acknowledged beamforming algorithm available in literature. A detailed scrutiny of Capon also referred MVDR beamforming technique has been done in [2]. The particular design principles for MVDR explained in literature[1] to [10] include the generalized sidelobe canceler [3,4,5,7] the design principles that use the worst case optimization and the outage probability constrained optimization,one-dimensional and muli-dimensional covariance fitting, eignvalue beamforming using a multi-rank MVDR beamformer and subspace selection, and steering vector estimation with as little as possible prior information [8-10]. This research aims at exploring the diagonal loading principle and eigenspace projection principle.

II.PROBLEM FORMULATION AND PROPOSED METHODS

When steering vector error exist, we consider the general solution where mismatch is additive random perturbation. Then random steering vector error $\boldsymbol{V_r}$, given by

$$V_r = S_s - S_d$$

$\boldsymbol{S_s}$ represents the phase vector of the desired signal and $\boldsymbol{V_r}$ denote the Gaussian random vector with mean zero. Steering vector can be calculated by [7]

Minimizing $w^T R_z w$ subject to $w^T S_d = 1$ (1)

$and\ w^T R_z w \ =E\{|y(t)|\}=E\{|Z(t)|\}$ represents the output power of the signal and $R_Z = E\{Z(t)z^T(t)\}$, The script R represents the complex conjugate transpose.The optimal solution of (**1**) and output power of the signal is given below

$$W_O=R_Z^{-1}S_d/S_d^T R_Z^{-1}S_d \qquad (2)$$

So,

$$E\{|y(t)|\} = 1/S_d^T R_Z^{-1}S_d \qquad (3)$$

The probability distribution function PDF [8], can be written

$$PDF(V_r) = [\pi^M \det(R_s)]^{-1}\exp\{-V_r^T R_S^{-1} V_r\} \qquad (4)$$

R_s is the covariance matrix. So the steering vector error can be defined as

$$f_1 = -(S_s - S_d)^T R_s^{-1}(S_s - S_d) \qquad (5)$$

To fix the problem of steering vector errors in the robust adaptive beam forming ,we discuss three methods,

1. First Method

By using the equations **(2)** and **(5),** we obtained the function regarding to steering vector error which is defined below as [9]

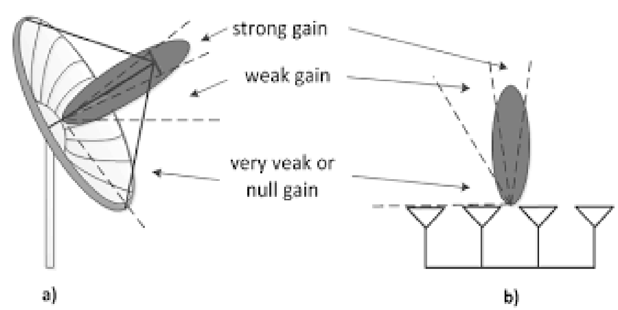

Fig.2,Beamforming with and without interference

$$J(s) = s^T R_Z^{-1} s + k(s - s_d) \qquad (6)$$

The equation **(6)** first term, represents the output power corresponding to the constraint vector **s** and second term of the equation represents a log likelihood function related constraint.**k** is the parameter [10] which represents the relative weight between these two terms.$\boldsymbol{s_o}$ is the optimal solution which is used for the desired steering constraint vector for robust adaptive beamforming.The optimal solution is given by

$$s_o = (I + \frac{1}{k} R_S R_Z^{-1})^{-1} S_d \qquad (7)$$

Substituting equation **(7)** into **(2)** we obtained the optimal Weight vector $\boldsymbol{W_o}$ corresponding to the

$s_o$given below:

$$W_o = [s_d^T(I + \frac{1}{k}R_z^{-1}R_s)_z^{-1}R_z^{-1}(I + \frac{1}{k}R_sR_z^{-1})^{-1}S_d]^{-1}.(R_z + \frac{1}{k}R_s)^{-1}S_d. \quad (8)$$

The steering vectors are obtained via minimization [11] of the expected prediction risk:

$$W_o = argminE\|W^TX - S\|_2^2 \quad (9)$$

However, the error which are occurred in the beamforming such as steering vector error caused by the mismatch of arrival,due to the array calibration especially high signal-to-noise ratio **(SNR)**.The eigenanalysis is used to remove the component of signal of interest **(SOI).**

2. The Second Method

The eigenvalues are used for interference cancellation and to achieve the optimal **(SOI).**To obtaine the desired signal and to remove the interference a spatial filter[12], a new covariance matrix is employ as (M-1)*(M-1).

$$R_B = E\{Bxx^TB^T\} \quad (10)$$

Where **B** is a matrix of dimension (M-1)*(M-1) and of rank M-1, All the row of the vector B are orthogonal to the steering vector of SOI.

By eigen- decomposition of R_B to BB^T,Then we get $\boldsymbol{E_r} = [\boldsymbol{e_1}, \boldsymbol{e_2}, \dots\dots, \boldsymbol{e_q}]$ which composed of eigenvectors corresponding to the q dominant generalized eignvalues. The weight vector of the Eigenanalysis interference canceller (EIC) [13] can be written as:

$$E_{EIC} = \frac{P_I^{\perp}a_b(\theta_0)}{a_b^T(\theta_0)P_I^{\perp}a_b(\theta_0)}, \quad (11)$$

Where $P_I^{\perp} = I - R_1(R_1^TR_1)^{-1}R_1^T, R_1 = BB^TE_r$, and $a_b(\theta_0)$ is the vector that consist of M-1 elements of $a(\theta_0)$.

Eigenvalues of the optimal weight vector can be written as:

$$W_O = \mu[E_sA_s^{-1}E_s^T + E_nA_n^{-1}E_n^T]a(\theta_o), \quad (12)$$

Where $\mu = 1/(a^T(\theta_o)R^{-1}a(\theta_o))$.Orthogonality between SOI and noise signal=0,so the optimal weight vector can be simplified as [14]:

$$W_{ESB} = \mu[E_sA_s^{-1}E_s^T]a(\theta_o) \quad (13)$$

And weight vector is computed by as:

$$W = \mu R^{-1}a(\theta_o). \quad (14)$$

3. Third Method

Due to the uncertainty set of steering vector [15], the Robust Beamforming can be calculated as:

$$min_a = a^TR_{xx}^{-1}a \ subject\ to\ \|a - a(\theta_1\|^2 \leq \beta, \quad (15)$$

So, a is the estimate of the desired steering vector and β is the uncertainty level. If want to prevent the solution a=0,so $\beta < \|a(\theta_1)1\|^2$.

By using Langrang multiplier, obtained spherical constraint equation so,

$$g(\lambda) = \|(I + \lambda R_{xx}.1\|^2 = \beta \quad (16)$$

The output of the steering vector to the signal can be defined as:

$$P = 1/b^TR^{-1}b \quad (17)$$

So, b represents steering vector,but differ from the true steering vector[16]. The SINR at the output of the beamformer when it is steered to the signal can be written as:

$$SINR = P_{s+I+N} - \frac{P_{I+N}}{P_{I+N}} \quad (18)$$

P_{s+I+N} is the output power of combined interference and noise ,P_{I+N} is the output power when the signal is absent and the final equation is below,

$$P_{s+I+N} = \frac{1}{b^TR^{-1}b} \quad (19)$$

$$P_{I+N} = 1/b^TR^{-1}b \quad (20)$$

III. SIMULATION AND COMPUTER RESULTS

3.1: Output SINR versus SNR Mismatch

DOA mismatch is uniformly distributed on [0,7],and the actual angle of arrival is 0^{o}. The number of snapshot is N=60

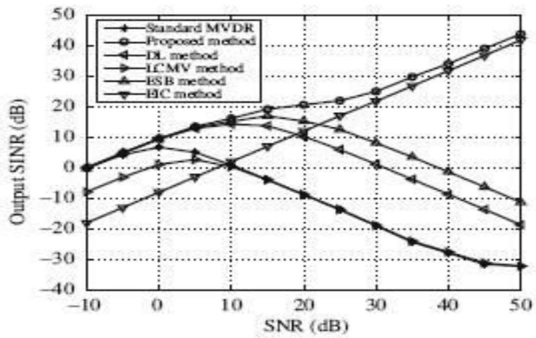

Fig.3 output SINR versus SNR for steering vector mismatch

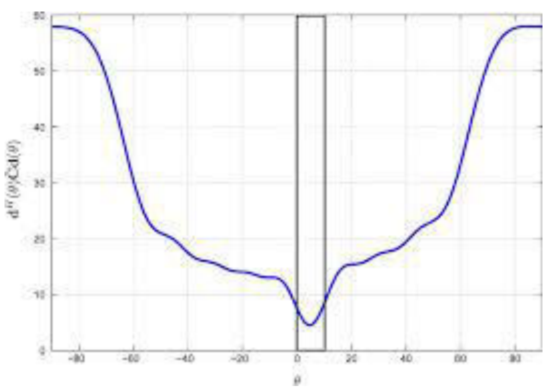

Fig.4,steering vector with different angle

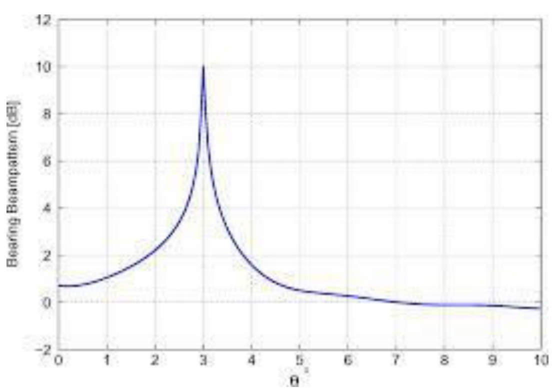

Fig.5, RAB based on steering vector estimation

IV. CONCLUSION:

The particular design principles for MVDR RAB include the sidelobe canceller approach, the worst-case optimization and the outage probability constrained optimization, one-dimensional and multi-dimensional covariance fitting, eigen value beamforming using a multi-rank MVDR beamformer and subspace selection, and steering vector estimation with as little as possible prior information. Robust beam forming algorithm that could mitigate array calibration errors. More, the existing techniques that address the stated problem would be analyzed. Performance comparison of the investigated techniques will also be carried out by selecting appropriate performance metrics.

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